Intrafaculty Colloquium: Mathematics and Physics

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Chern-Weil theory for quasi-isomorphisms

Chern-Weil theory was used by Shulman (Berkeley thesis, 1972) to give explicit closed simplicial differential forms on the simplicial manifold $B_\bullet\mathrm{GL}(n)$, which realize the Chern classes. For example, the only nonzero component of the differential form c_1 is

$$\mathrm{Tr}(g^{-1}dg) \in A^1(\mathrm{GL}(n)),$$

and the fact that it defines a closed simplicial form on $B_\bullet\mathrm{GL}(n)$ amounts to the multiplicativity of the determinant. Likewise, there are two nonzero components of c_2, namely

$$\tfrac{1}{6}\mathrm{Tr}((g^{-1}dg)^3) \in A^3(\mathrm{GL}(n))$$

and

$$\tfrac{1}{2}\mathrm{Tr}((g_1^{-1}dg_1)(dg_2g_2^{-1})) - \mathrm{Tr}(g_1^{-1}dg)\mathrm{Tr}(g_2^{-1}dg_2) \in A^2(\mathrm{GL}(n)\times\mathrm{GL}(n)),$$

and the fact that the simplicial differential form that these forms comprise is closed is known as the **Polyakov-Wiegmann formula**.

In this talk, I will define an extension of these forms to the classifying stack of perfect complexes (that is, extend these formulas to the case where g is a quasi-invertible map between finite-dimensional complexes). These extensions were proved to exist by Toën and Vezzosi, but the explicit formulas are new. In the case of c_1, we obtain a new perspective on the determinant of quasi-invertible maps, defined by Knudsen and Mumford.

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